

# Density of states of $s+d$ -wave superconductor with Anderson impurities

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**Abstract.** We present results for the density of states of a  $s+d$ -wave superconductor containing finite concentration of Anderson impurities within the self-consistent slave boson approximation. There may be zero, one or two peaks in the energy gap at low energies. The height of the peaks is controlled by the impurity concentration whereas their position depends on the strength of interaction between impurities and the conduction band. Experimental consequences are briefly discussed.

The existence of a superconducting order parameter of mixed symmetry is frequently discussed both on theoretical and experimental grounds. While the  $d$ -wave order parameter in many of the high- $T_c$  materials is widely agreed upon there is an intriguing possibility for the appearance of a subdominant  $s$ -wave component. Some tunneling measurements in overdoped  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  show the splitting of zero-bias conductance peak.[1] This may be interpreted as evidence for a mixed-gap state breaking the time-reversal invariance such as  $s + id_{x^2-y^2}$  or  $d_{xy} + id_{x^2-y^2}$ . [2] Some magnetic penetration depth measurements in single crystals of  $\text{YBa}_2\text{Cu}_4\text{O}_8$  also suggest simultaneous existence of both  $s$  and  $d$ -wave components of the order parameter.[3]

The additional component of order parameter may be a result of proximity to interfaces,[4, 5] magnetic impurities[6] and effects related to magnetic fields. In high- $T_c$  compounds the  $s$ -wave component may appear as a consequence of the orthorhombic distortion of the tetragonal crystal lattice structure.

Our aim is to show that the low-energy behavior of a superconductor with magnetic impurities depends very sensitively on details of the model. Therefore in some experimental settings magnetic impurities may be used as a tool to probe the nature of such mixed state.

We consider the order parameter of  $s + d$  symmetry on a cylindrical Fermi surface

$$\Delta_s + e^{i\theta} \Delta_d(\hat{k}), \quad (1)$$

where  $\Delta_s$  and  $\Delta_d$  are amplitudes of  $s$ - and  $d$ -wave component respectively. We fix  $\theta = 0$ , leaving more general analysis for a future work. Our model consists of a BCS-like term in the Hamiltonian and the Anderson impurity treated in the self-consistent large- $N$  mean field approximation.[7] The  $\tau_0$  and  $\tau_1$  components of self-energies for the conduction electrons and the impurity have the following form,

$$\tilde{\omega} = \omega + \frac{nN}{2\pi N_0} \frac{\bar{\omega}}{-\bar{\omega}^2 + \epsilon_f^2 + \bar{\Delta}}, \quad (2)$$

$$\tilde{\Delta} = \Delta_s + \frac{nN}{2\pi N_0} \frac{\bar{\Delta}}{-\bar{\omega}^2 + \epsilon_f^2 + \bar{\Delta}}, \quad (3)$$

$$\bar{\omega} = \omega + \Gamma \left\langle \frac{\tilde{\omega}}{(\tilde{\Delta}^2(k) - \tilde{\omega}^2)^{1/2}} \right\rangle, \quad (4)$$

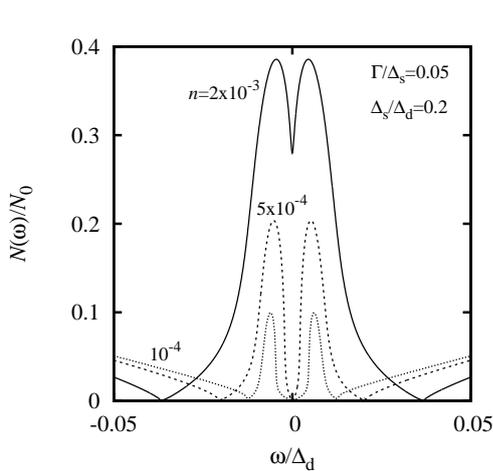
$$\bar{\Delta} = \Delta_s - \Gamma \left\langle \frac{\tilde{\Delta}(k)}{(\tilde{\Delta}^2(k) - \tilde{\omega}^2)^{1/2}} \right\rangle, \quad (5)$$

where  $\langle \rangle$  denotes the Fermi surface average.

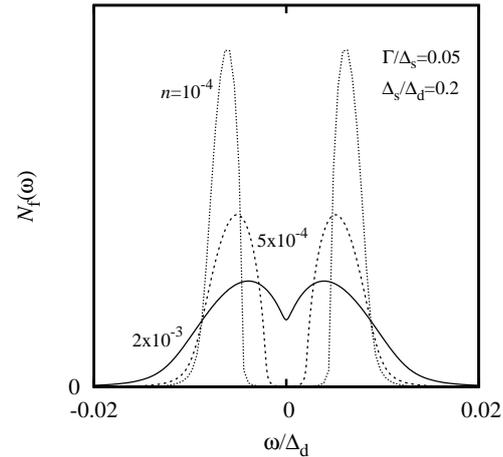
We present results for the density of states (DOS) of such system, assuming the relative magnitude of the order parameter components  $\Delta_s/\Delta_d = 0.2$ . We also assume  $\Delta_d/D = 0.01$ , where  $2D$  is the bandwidth of the conduction electron band.

For hybridization  $\Gamma \ll \Delta_s$  and resonant level energy scale  $\epsilon_f \ll \Delta_s$ , and for small concentrations there are two peaks near the gap center, see figure 1. As impurities are added to the system the two peaks merge into one peak centrally located at the Fermi energy.

Figure 2 shows the impurity spectral function for the parameters of figure 1.



**Figure 1.** Conduction electron DOS for  $\Delta = \Delta_s + \Delta_d \cos(2\phi)$ , where  $\Delta_s/\Delta_d = 0.2$  and  $\Gamma/\Delta_s = 0.05$  for impurity concentration  $n = 10^{-4}, 5 \times 10^{-4}, 2 \times 10^{-3}$  and  $\epsilon_f = 0$ .  $N_0 = 1/2D$  is the DOS of a simplified flat conduction band.

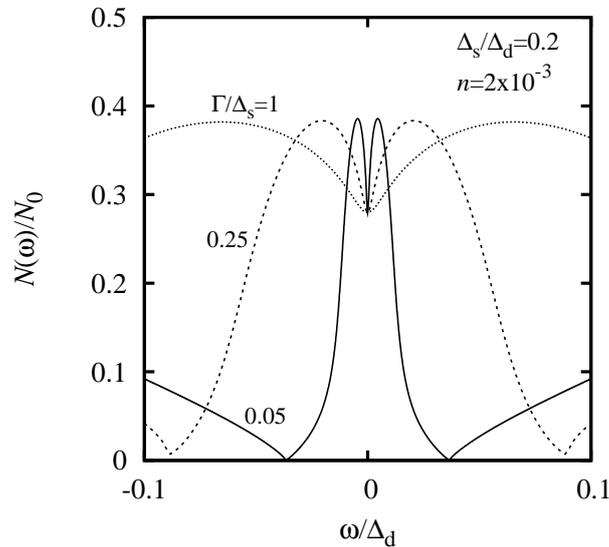


**Figure 2.** Impurity spectral function for the same parameter set as in figure 1.

For  $\Delta_s/\Delta_d \ll 1$  the critical impurity concentration  $n_0$  for the transition between between the exponentially small DOS at the Fermi energy  $N(0)$  and  $N(0)/N_0 \simeq \Delta_s/\Delta_d$  is approximately proportional to  $(\Delta_s/\Delta_d)^2$ . For  $n < n_0$ ,  $N(0)/N_0 \sim (\Delta_s/\Delta_d) \exp(-\alpha(\Delta_s/\Delta_d)^2/n)$ , where  $\alpha$  is a numerical factor. For  $n > n_0$ ,  $N(0)$  quickly approaches the DOS of a  $d$ -wave superconductor. These relations are valid when the resonant level position is the smallest energy scale,  $\epsilon_f \ll \Gamma, \Delta_s$ .

The difference between magnetic and nonmagnetic impurities in a superconductor with  $\Delta_s/\Delta_d \ll 1$  lies in the character of the transition from the exponentially small  $N(0)$  to finite  $N(0)$  in the strong scattering limit. In the case of magnetic impurities there are two peaks on both sides of  $n_0$ . The transition is discontinuous for  $\epsilon_f \ll \Gamma, \Delta_s$ . For  $\epsilon_f > \Delta_s$  there is a smooth crossover between the two regimes of DOS.

The increase of  $\Gamma$  moves the resonant-level peaks away from the gap center, see figure 3. The peaks vanish into the shoulders of the superconducting gap for  $\Gamma \gg \Delta_s$ .



**Figure 3.** Impurity spectral function for fixed impurity concentration and varying hybridization  $\Gamma$ .

We expect the qualitative conclusions of this work to be generally valid for order parameters with lines of nodes and a nonvanishing Fermi surface average. The obtained DOS implies some obvious consequences for the thermodynamic and transport properties. The low- $T$  and low-energy properties may develop peaks in presence of magnetic impurities. These peaks are sharper than analogous features in a  $d$ -wave superconductor because finite  $\Delta_s$  reduces the width of the impurity resonance. Experiments at low concentration of magnetic impurities may be a useful test of the existence of the  $s$ -wave component in unconventional superconductors.

Impurities strongly influence the relative stability of various superconducting states.[8] The minimum free energy for a BCS-like superconductor is likely to be obtained for a  $s + id_{x^2-y^2}$  state.[9] However the  $s + d$ -state might still be realized under some conditions. Results for  $s + id$  state and discussion of differences between  $s + d$  and  $s + id$  states will be presented in a separate work.

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### References

- [1] Dagan Y and Deutscher G 2001 *Phys. Rev. Lett.* **87** 177004
- [2] Sharoni A, Millo O, Kohen A, Dagan Y, Beck R, Deutscher G and Koren G 2002 *Phys. Rev. B* **65** 134526
- [3] Khasanov R, Shengelaya A, Karpinski J, Bussmann-Holder A, Keller A and Müller K A 2008 *J. Supercond. Nov. Magn.* **21** 81 and references therein
- [4] Matsumoto M and Shiba H 1995 *J. Phys. Soc. Jpn* **64** 3384
- [5] Sigrist M 1998 *Prog. Theor. Phys.* **99** 899
- [6] Balatsky A V 1998 *Phys. Rev. Lett.* **80** 1972
- [7] Borkowski L S 2008 *Phys. Rev. B* **78** 020507
- [8] Florens S and Vojta M 2005 *Phys. Rev. B* **71** 094516
- [9] Musaelian K A, Betouras J, Chubukov A V and Joynt R 1996 *Phys. Rev. B* **53** 3598